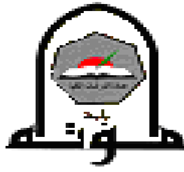


Mu'tah University
Deanship of Graduate Studies

The Fourier Transform And Its Applications

By

Monther Awny Qablan

Supervised by

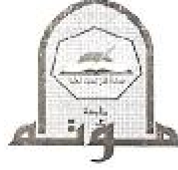
Prof. Rateb Al-Btoosh

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'ourier Transform and its Applications

استكمالاً لمتطلبات الحصول على درجة الماجستير في الرياضيات.

القسم: الرياضيات والأحصاء.

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Dedication

ء ادهلأا

قاسينا اكثر من هم وعانينا الكثير من الصعوبات وها نحن اليوم نطوي سهر الليالي وتعب الايام وخلاصة مشوارنا بين دفتي هذا العمل المتواضع.

الى من سعى وشقى لأنعم بالراحة والهناء الذي لم يبخل بشيء من أجل دفعي في طريق النجاح الذي علمني ان ارتقي سلم الحياة بحكمة وصبر الى والدي العزيز.

الى الينبوع الذي لا يمل العطاء الى من حاكت سعادتي بخيوط من يدها الى والدتي الغاليه.

الى من حبهم يسري في عروقي ويلهج بذكراهم فؤادي الى اخواني وأصدقائي.

الى من علمونا حروفا من ذهب وكلمات من درر وعبارات من احلى واسمى العبارات الى اساتذتنا الكرام.

الى روح صديقي واخي ورفيقي عمري مراد محمود غرايبه ومحمد عزام القبلان.

اهدي هذا العمل المتواضع.

منذر عوني قبلان

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Monther Awny Qablan

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ABSTRACT

The Fourier Transform And Its Applications

Monther Awny Qablan

Mu’tah University, 2012

In this thesis, first we studied the Fourier series for a limited interval and under the condition that the function is sufficiently smooth and periodic, then we defined Fourier transforms of all types of functions whether it’s periodic or nonperiodic. Then we studied it’s properties, it’s relationship with derivatives and some important theories in a consecutive and easy way. Next, we got the result depending on these properties. Finally, we studied the result of Schwartz space in which we studied the properties and some theories task in this space.

الملخص

تحليلات فورير وتطبيقاته

منذر عوني قبلان

جامعة مؤتة، 2012

في هذه الاطروحه قمنا بدراسة متسلسلات فوريير على فترة محدودة وتحت شرط ان يكون الاقتران دوري، ومن ثم قمنا بتعريف تحويلات فوريير على اي اقتران (دوري اوغير دوري)، ودرسنا خصائصه وعلاقته مع المشتقات وبعض النظريات المهمه باسلوب متسلسل وسهل، وحصلنا على نتائج بالاعتماد على هذه الخصائص، ومن ثم درسنا هذه النتائج تحت شرط فضاء شوارتز وقمنا بدراسة خصائصه وبعض النظريات المهمه في هذا الفضاء.

Chapter One

Introduction

Fourier series and generalization to Fourier transform have become an essential part of the scientific branches, engineers and mathematical from both an applied and theoretical point of view.

Fourier transform play an important part in the theory of many branches of science. While they might be regarded as purely mathematical functional as is customary in the treatment of other transforms, they also assume in many fields just as definite a physical meaning as the functions from which they stem.

The motivation for the Fourier transform comes from the study of Fourier series. In the study of Fourier series, complicated functions are written as the sum of abstract waves mathematically represented by sines and cosines. Due to the properties of sine and cosine it is possible to recover the amount of each wave in the sum by an integral. In many cases it is desirable to use Euler's formula, which states that $e^{i\vartheta} = \cos\vartheta + i\sin\vartheta$, to write Fourier series in terms of the basic

waves $e^{i\vartheta}$. This has the advantage of simplifying many of the formulas involved and provided a formulation for Fourier series that more closely resembles the definition followed in this thesis. This thesis from sines and cosines to complex exponentials makes it necessary for the Fourier coefficients to be complex valued. The usual interpretation of this complex number is that it gives both the amplitude (or size) of the wave present in the function and the phase (or the initial angle) of the wave. This passage also introduces the need for negative "frequencies". If ϑ were measured in seconds then the waves $e^{i\vartheta}$ and $e^{i\vartheta}$ would both complete one cycle per second, but they represent different frequencies in the Fourier transform. Hence, frequency no longer measures the number of cycles per unit time, but is closely related.

There is a close connection between the definition of Fourier series and the Fourier transform for a functions f which are zero outside of an interval. For such a function we can calculate its Fourier series on any interval that includes the interval where f is not identically zero. The Fourier transform is also defined for such function. As we increase the length of the interval on which we calculate the Fourier series, then the Fourier series coefficients begin to be similar the Fourier transform and the sum of the Fourier series of f begins to look like the inverse Fourier transform.

Any periodic function can be expressed as the sum of series of sines and cosines (of varying amplitude). Fourier series can be generalized to complex number and further generalized to derive the Fourier transform. Fourier transform maps a time series (e.g audio samples) into the series of frequencies (their amplitudes and phases) that composed the time series. Inverse Fourier transform maps the series of frequencies (their amplitudes and phases) back into the corresponding time series.

We can say that the Fourier series of periodic function associated with a sequence of numbers, (namely the Fourier coefficient), to that a

function on $\mathbb{R}$, the analogous object associated to will in fact be

another function on ℝ which is called the Fourier transform of .

Since the Fourier transform of a function on ℝ, one can observe symmetry between a function and its Fourier transform, whose analogue is not as apparent in the setting of Fourier series.

In summary, we studied the properties of Fourier Transforms according to the conditions of Schwartz space and added some important applications on Fourier Transform and Schwartz Space.

Chapter Two

Preliminaries and Basic Concepts

2.1 Definitions and Preliminaries

Definition 2.1.1 A function is said to be piecewise continuous on an interval if it is define and continuous except possibly at finite number of points, a and at each points of discontinuity the left and right hand limits

)

exist. Note that we do not require that is defined at all if is define it does not necessarily equal either the left or the right hand side.

Definition 2.1.2 A sequence of functions defined on an interval converges pointwise to a function if for each , the numerical sequence converges to . We write pointwise on I, as .

The series pointwise on an interval if for each , .

Definition 2.1.3 A sequence of functions is said to be convergent uniformly to a function on a subset if for every there exist an integer such that and all.

Theorem 2.1.1 If convergent uniformly and each is continuous then is also continuous function.

Theorem 2.1.2 Let $\subset \mathbb{R}$, suppose the functions are bounded by

where the are fixed positive constant, if the series is convergent, then the series

is convergent uniformly and absolutely to a function , for all

Definition 2.1.4 Let be any real number, , and the vector space of all complex valued function of all real variable x, -, such that is lebesgue measurable, and

we call the number the -norm of .

Definition 2.1.5A piecewise continues function , defined on an interval , is integrable (or the class or simply) on I if the integral

is finite.

The norm of a function is defined by

Definition 2.1.6 A piecewise continuous function defined on an interval is squared – integrable (or of class or simply on), if the integral is finite.

The norm of a function is defined by

Definition 2.1.7 A piecewise continuous function defined on an interval is bounded (or) on , if there is a number such that , for all .

The -norm if a function is defined by

Definition 2.1.8 A complex valued function is called square-integrable on the interval , if it satisfies

Theorem 2.1.3 (Cauchy-Schwartz inequality)

Let and be on the interval . Then

Definition 2.1.9 Let (x) be a norm vector space. Then a sequence is said to be converge in norm

Definition 2.1.10 Given , we say that a function defined on an interval I is on I if it is -times continuously differentiable on I. on I means that is continuous on I. is on I if it is on I for every .

We say that is on I if it is on I and compactly supported, on if it is on I and compactly supported, and on if it is on and compactly supported.

Definition 2.1.11 The sequence converges in to the function if

or

the series

if the sequence of partial sums

Definition 2.1.12 A function is periodic of period , if there is a number such that , if there is such a then the smallest one for which the equation holds is called the fundamental period is also a period.

Definition 2.1.13 A function defined on ℝ, is said to be moderate

decrease, if is continuous and there exists a constant so that

we shall denote by the set of moderate decrease on ℝ.

We note that whenever belongs to , then we may define

Proposition 2.1.1 The integral of a function of moderate decrease satisfies the following properties:

- Linearity: if and are constants then

- Translation invariance: for every we have

- Continuity : if , then

Definition 2.1.14 Let be a vector space over a scalar field, a norm on is a function such that for all , and:

- and if and only if.
- .
- (Triangle inequality).

In addition, the pair is called a norm vector space.

Lemma 2.1.1 (Uniform Boundedness Principle)

If is a norm vector space, and is a Banach space, and is a collection of bounded linear maps for each (where Δ is any index set not necessarily countable) then either:

- or,
- There exist such that

Theorem 2.1.4 Fubini's theorem

Suppose that and are completed measureable spaces, and is measurable, if . Then

Corollary 2.1.1 If for some function and h then

2.2 FOURIER SERIES

Let be the inner product space of Riemann square-integrable function on the interval , here the inner product is

(This satisfies the condition provided we identify all function with the same integrals). Thus, from now on our function will be assumed - periodic.

Consider the set

for it is easy to check that , are on set in .

Theorem 2.2.1 “Bessel’s inequality”

Let be a function defined on such that has a finite integral on , if and are the Fourier coefficients of the function then we have

In particular the series is convergent.

Definition 2.2.1 Given . The Fourier series of is the projection of on

in terms of sines and cosines this is usually written as

with

Where

with Bessel inequality

Definition 2.2.2 A Fourier series with finitely many nonzero terms is

called a trigonometric polynomials and written

(We say such a has degree N) since

also written   in a more elegant complex form

Theorem 2.2.2 Supposes   is continuous -periodic and piecewise   then the series are Uniformly converges on $\mathbb{R}$, and ,

where

Theorem 2.2.3 (Parseval's equality) (convergent on the norm)

Let . Then

This is equivalent to the statement that (are on basis for.

Theorem 2.2.4 Suppose is periodic with period   is piecewise continuous on , and is piecewise continuous on Then the Fourier series of is converges to

Lemma 2.2.1 If is periodic, then its integral is periodic, if and only if so that has mean zero on the interval .

Example 2.2.1 Expand   ,   in Fourier series?

Using the expansion of the function and,

we find that

We can use this example to calculate the sum of some important trigonometric series for immediately gives

and from (2.2.1) and (2.2.2) we infer that

Since the terms of the series on the left do not exceed in an absolute value the series is uniformly convergent, which means that its sum is continuous for all. Therefore (2.2.5) is valid for and not just for

Similarly we have by Fourier series

gives

similarly we have by Fourier series

for

and by Fourier series

and gives

and by Fourier series for

gives

Moreover, subtracting (2.2.11) from (2.2.4), we obtain

and subtracting (2.2.12) from (2.2.5), we obtain

These formulas also allow us to calculate the sums of some numerical series, for example, if we set , (2.2.5) and (2.2.9) become

while if we set (2.2.11) becomes

2.3 change of scale

So far, we have only dealt with Fourier series on the standards interval of length (we choose for convenience, but all of the results and formulas are easily adapted to any other interval of the same length since physical object like bars and strings don't all come in this particular

length we need to understand how to adept the formulas to more general intervals.

The basic idea is to rescale the variable so as to stretch or construct the standard interval.

Any symmetry interval of length can be rescaled to the standard interval by using the linear change of variables so that whenever .

Given a function defined on the rescaled function

lives on .

Let

be the slandered Fourier series for so that

Then reverting to the uncalled variable, we deduce that

The Fourier coefficient , can be computed directly from .

Indeed replacing the integration variable in and noting that

we deduce the

for the Fourier coefficient of on the interval

Theorem If the Fourier coefficient satisfy then the Fourier series is converges uniformly to the continuous function have the same Fourier coefficient

Theorem (Bessel's inequality) [21]

If is finite, then

Bessel's inequality can be proved easily, In fact we have

Multiply by the last integral above, and making use of equations

we obtain

Thus for all

and Bessel's inequality follows by letting, Bessel's inequality has a physical interpretation, if has finite energy in this sense that the right side of 2.3.1 is finite, then the sum of the modulo-squared of the Fourier coefficient is also finite.

We shall see that the inequality in equality (2.3.1) is actually an equality which says that the sum of the modulo squared of the Fourier coefficient is precisely the same as the energy of.

Because of Bessel's inequality it follows that holds whenever is finite.

The Riemann-Lebesgue lemma says that equation (2.3.1), holds in the following more general case .

Lemma 2.3.1 (Riemann Lebesgue lemma)

If is finite then equation (2.3.1), holds.

We use the following for the Fourier coefficient of a -periodic function

Lemma 2.3.2 (Riemann – Lebesgue)

Assume that is a -periodic bounded and integrable, then .

Chapter three

Relations between Fourier series and Fourier transformation

As we have seen, any (sufficiently smooth) function that is periodic can be built out of sins and cosines. We have also seen that complex exponential may be used in place of sin's and cosine's. We shall now use complex exponential, because the lead to less written and simpler computation, but yet can easily be converted into sin's and cosine's.

If has periodic its complex Fourier series expansion is

Not surprisingly each term in this expansion also has periodic because

we now develop an expansion for non-periodic function by allowing complex expansions (or equivalently sin's and cosine's) of all possible

periods, not just for some fixed so from now on, We do not assume that is periodic .

For simplicity we will only develop the expansion for function that are zero for all sufficiently large .

With a little more work, one can show that our conclusion apply to a much broader class of functions.

Let be sufficiently large that for all we can get a Fourier series expansion for the part of with by using the periodic extension trick.

Definition 3.1.1 Let be the unique function determined by the requirements that:

- ,
- is periodic of period

then for .

where

If we can somehow take the limit as we will get a representation of that is valid for all , not just those in some finite interval , this is exactly what we shall do, by the simple expedient of interpreting the sum in (3.1.2), as a Riemann sum approximation to a certain integral. For each integer, define the frequancy to be and denote by the spacing between any two successive frequencies. Also define

in this notation

for any as we let the restriction dissapear and the right hand side converges exactly to the integral

To see this cut the domain of the integral:

0

Small slices of width and approximate as in the above figure, the area under the part of the graph of , with between and by the area of rectangle of base and height this approximates the integral

by the sum

The approximation gets better and better so that the sum approaches

the integral so taking the limit of (3.1.3) as gives

The function is called the Fourier transformation of is to be thought of as frequencies profile of the signal.

Thus, once the Fourier transform of is defined as given above.

Accordingly, some authors define Fourier transforms as follows

where is used as the transform variable. Sometimes the letter or is used as the transform variable

Example 3.1.1 Rectangle function

Find the Fourier transform of rectangle function?

then

and similarly if

For the Fourier transform we compute (using integration by parts, and the factoring trick for the sine function). Then the .

Chapter four

Fourier transform and Schwartz Space

The theory of Fourier series and integrals has always had major difficulties and need a large mathematical apparatus in dealing with the questions of convergence. It engendered the development of methods of summation, although these do not lead to a completely satisfactory solution of the problem. For the Fourier transform, the introduction of distribution (hence the space) is inevitable either is an explicit or hidden form. As a result one may obtain all that is desired from the point of view of the continuity and inversion of the Fourier transform. The theory of Fourier series applies to function on the circle, or equivalently,

periodic function on $\mathbb{R}$.

4.1 properties of Fourier transform

In this section we begin by some properties of Fourier transform. If and any real number we define the Fourier transform of by

and say that is the Fourier transform of .We write symbols

In the special case when is even for all real values of take the form

If is odd for all real values of take the form

Now we give some basic properties of Fourier transform of a function in .

Property 4.1.1 If, then is bounded on , since for all real we have

where denotes the of   so that

Property 4.1.2 If   , then is continous on for if h is a real number then

and hence () is continuous at the point where

Property 4.1.3 If and are two constants and we form a new function as alinear compination of two old functions and then the Fourier transfom of is

Property 4.1.4 Let h be a fixed real number and then the Fourier transforms of the transform of by h equals

Since

then

Clearly that

Property 4.1.5 If we form a new function by scaling time by factor if:

- Let c , then the Fourier transform of is

- Let then the fourier transform of is

Since then

In general

Property 4.1.6 If we build a new function by differentiating old function then the Fourier transform of h is

now integrate by parts with , so that

assuming that then

This process can be repeated to see that if are all Fourier transform with all having zero limits as , then

Property 4.1.7 (Parseval's Relation)

The energy carried by a signal is

substitution in that

we have that

the formula

is called Parseval's Relation.

Property 4.1.8 (Duality)

The Duality property says that if we build a new time – domain function by exchanging the rules of time and frequency then the Fourier transformation of is to verify this just write down just the definition of and the Fourier inversion formula .

For and in both integrals make change of variables that the integration variable is.

So which is given by the last integral of the first line is exactly times the last integral of the second line with replaced by which is

Property 4.1.9 If denotes the complex conjugate of and then the Fourier transformation of is since the complex conjugate of

Property 4.1.10 If and for

and

then we have

Uniformly for in effect

Property 4.1.11 If then

This is called the composition rule. To prove it, note that

Using the fact that

and by Fubini's theorem, the integral

is symmetric in and so that we can interchange and

4.2 The inverse Fourier transform and convolution theorem

The inverse Fourier transform is defined and Fourier inversion to the integral we have just come up with can stand it is own as "transform", so we define the inverse Fourier transform of a function to be

The Fourier transform and are the same except for the minus sign in

the exponential in words we can say that if you replace by in the formula for the inverse Fourier transform then you are taking the Fourier transform that is

Example 4.2.1 Here's an example of how duality is used we know that

and hence that by duality we can find

.

Then .

Let's apply the same argument t find, Recall that is atriangle function we know that

but then

and since is even

Properties of the inverse Fourier transformation: In many cases the properties of the Fourier transform like corresponding properties of the Fourier transform with similar proofs.

Often the Fourier transform property is actually simpler because no boundary terms at arise in the integration. The property of linear is more or less obvious.

In the following development of the properties of the Fourier transform we will use the symbols or whichever is more convenient. The second is usually preferable if is given by a complicated formula.

Property 4.2.1

Property 4.2.2 [17]

Property 4.2.3 [17]

Property 4.2.4 [17]

Property 4.2.5[17] (

Property 4.2.7[17] If is real valued then

Note that since is real.

Convolution theorem The convolution of two functions and is the function

we use the notation

Theorem 4.2.1 (Convolution theorem)

in other notation if

then

By the symmetry between the Fourier transform and its inverse we can say that we have

Lemma 4.2.1 We write for convenience

Now make the change variable in the inner integral then and the limits are the same the result is

then the outer integral produces the Fourier transform of

Replace the variable name for from to .

We have discovered that the signal and are combined into a signal

In other words

4.3 Differentiability of Fourier transforms

In this section we discuss some properties of differentiability of Fourier transform and we generalize the results that are stated by many other authors, and in this section and section we give some propriety in Shwartz space using the main result.

Another basic property of the Fourier transform is how it behaves in relation to differentiation is the shorthand way of describing the situation.

We know how to differentiate a distribution, and it's an easy step to bring the Fourier transform into the picture we'll then use this to find the Fourier transform for some common functions that therefore we have not been able to treat.

Proposition 4.3.1 Fourier transform of . Assume that and are both such that their Fourier transform are define we have

Proposition 4.3.2 Fourier transforms for If and as then their integration by parts gives

Proposition 4.3.3 If the Fourier transforms of and are both defined and then their integration by parts gives.

This process can be repeated to see that if are all Fourier transform with all having zero limits as then

Theorem 4.3.1 Let and , where is a positive integer then is continuously differentiable times for and we have

So that

Proof We are going to use induction on , we assume that the result is true for all such that

Let

also let

then

now

So pointwise, for almost every, as and

note that

Let .By applying the Mean value Theorem on

We have

then

We derive from (4.3.4) and by hypothesis that

Hence

By Riemann-Lebesgue Theorem on dominated convergence, we

conclude that in the is

By property we get

The last integral is a bounded continuous function of.

Now

Hence

This shows that the result is true for any positive integer .

Therefore

However

Theorem 4.3.2 If , is continuously differentiable times , for , then

So that

Proof Let be real number, For , we have

Now tends to finite limits as, then

So that

But hence letting in, we have

by the same argument, we have

which implies that

This hold also for by continuity (since the left hand side is zero and the right hand side is

Now

By Riemann Lebesgue and by (4.3.9), we conclude that,

since by assumption.

Theorem 4.3.3 If , then the integral

defines a continuous function of

Proof

Then consider

by

and

For almost all it follows from Riemann Lebesgue theorem on dominated convergence that.

and hence is a continuous function at the point wehere

Remark 4.3.1 We indicate that if it does not necessary that the Fourier transform belongs to, we can see this in the following example.

Taking

It is clearly that but for any real number

,

we have

note that

Example 4.3.1 We solve Laplace equation

subject to the boundary condition and as

.

Solution Taking the Fourier transforms of the equation with respect to we get

Solve

We get

By the condition as   then

So

Using (2) we get   so

now taking the inverse transforms for

now where

by convolution theorem

4.4 Schwartz space and differentiability of Fourier transform

The Schwartz space on $\mathbb{R}$ consist of the set of all indefinitely differentiable function so that and all derivatives ..., are rapidly decreasing in the sense that

Definition 4.4.1 The Schwartz space of all complex valued- function of real variable such that is differentiable infinitely often and for any integer where denoting the derivative of .

We denote this space by and should verify that is a vector space over , moreover, if we have

Proposition 4.4.1 If then is bounded and belongs to for any integer .

Proof Let . Then for any integer there is such that

By addition (4.4.1) and (4.4.2)

We have

Proposition 4.4.2 If then is bounded and belongs to for any integers

This expresses the important fact the Schwartz space is closed under differentiation and multiplication by polynomials.

The Fourier transform on a Schwartz space

The Fourier transform of a function is defined by

Some simple properties of the Fourier transform are gathered in the following proposition we use the notation to mean that denotes the Fourier transform of

Proposition 4.4.3 If then:

- whenever
-
- whenever
-
-

Proof Property (1) is an immediate consequence of the translation invariance of the integral and property (2) follows from the definition.

Also, the third the proposition if then

Integration by parts gives

so letting tends to infinity gives (4).

Finally, to prove property (5) we must show that is differentiable and find its derivative.

Let and consider

Since and are of rapid decrease there exist an integer So that

Implies

Hence for we have

Theorem 4.4.2 If then the Fourier transform belongs to .

Proof Let . Then by 4.4.1 of proposition president we have, for any integer, so that by 4.3.1 is differentiable, infinity often.

On the other hand, for are positive integers, then by theorem 4.3.1 we have

and by theorem 4.3.2, we have

Since from 4.4.1 and by Riemann Lebesgue theorem, we conclude that

Hence the proof is complete.

Proposition 4.4.4 There exists an element with and

Proof If, by propodition 4.3.1

then it is Fourier transform is Schwartz and conversely, we also know that

Thus we just have to arrange that and all its derivatives vanish at the origin.

We do know that there is a non trivial Schwartz function which vanishes outside the interval for instance.

Taking this at the Fourier transform of and when choosing the positive constant so that we get that

Example 4.4.1 A simple example of a function in is the Gaussian defined by which plays a central role in the theory of the Fourier transform.

The reader can check that the derivatives of are of the form where is a polynomial, and this immediately shows that . In fact, belongs to whenever

We will normalize the Gaussian by choosing we begin by considering the case because the normalization:

To see why (4.4.1) is true, we use the multiplication property of the exponential to reduce the calculation to a two dimensional integral. More precisely, we can argue as follows:

where we have evaluated the two-dimensional integral using polar coordinates

Note We notes that it's not necessary that any function rapidly decreasing belongs to the Schwartz space

Remark As a final remark, note that although decreases rapidly at infinity it is not differentiable at 0 and therefore does not belongs to

Lemma 4.4.1If then that is

Proof Since is rapidly, it suffices to prove this on the real line. As

Notice that

is a unique solution of

We show that also solves equation (4.4.1).

First notice that

We now compute

Thus

Proposition 4.4.5 If , then

Proof To prove this proposition, we need to digress briefly to discuss the interchange of the order of integration for double integrals. Suppose is a continuous function in the plane . We will assume the following decay condition on

Then, we can state that for each the function is of moderate decrease in y, and similarly for each fixed the function is of moderate decrease in Moreover, the function is continouos and of moderate decrease, similarly for the function finally.

We now apply this to , then and so

The multiplication formula and the fact that the Gaussian is its own Fourier transform lead to a proof of the first major theorem.

Proposition 4.4.6 Let and is such that and the Fourier transform is such that , then such that

Proof Since is given, we only need to find satisfying

and then will solve the problem.

If we set that , as we can with uniquely determine, then

so

where by definition is the function with Fourier transform equal to in by assumption.

Proposition 4.4.7 for all nonnegative integer number and then

and

Proof By proposition 4.3.3

We know that

Then by substitution (4.4.2) in (4.4.3) we get

We are going to use the norm of (4.4.4)

Since and hence by proposition 4.4.1 and closed for differentiable then the right hand side is finite and depends on and , we have shown that there is a constant with

Chapter five

Conclusion

Firstly, introduced in chapter two the Fourier series in a limited interval with a periodic functions, then we improved and extended the interval into similar interval under the condition of periodic function.

Next, we removed the periodic condition and extended the interval to , from here we defined Fourier transform. Fourier transform maps a time domain into the frequency domain and we defined the inverse Fourier transform maps a frequency domain back into the time domain.

Then, we studied in chapter three the Fourier transform and its properties in the Fourier transform, we prove its properties and theorem easily and clearly. In addition, we studied the properties of Fourier transform along with its relation with derivatives. We found a new relation depending on the properties of the derivative Fourier transform.

Then, in chapter four we studied Fourier transform with theorems in Schwartz space and the theorems of Schwartz space. We studied the behavior of derivative Fourier transform under the condition of Schwartz space. We proved different theorems in different way and new ways in

which we it could be understand and we added some important application to the Fourier transform and Schwartz space.

Finally, in chapter four we studied the Schwartz Space and its relationship to derivatives Fourier Transform and we got the important theories in relationship between the Schwartz Space and derivatives Fourier transform also we added some important applications on Fourier Transform and Schwartz Space.

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